JV-003-1015025 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

October – 2019

Physics : Paper - 501

(Mathematical Physics, Classical Mechanics & Quantum Mechanics)

(New Course)

Faculty Code : 003

Subject Code : 1015025

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.

(2) Symbols have their usual meaning.

(3) Figures on right hand side indicate full marks.

1 (a) Answer in brief : **4**

(1) Write Fourier coefficient a_0.

(2) If $f(x)\sin nx$ is odd function, then which type the function $f(x)$ is ?

(3) Sine series is also known as ___________ series.

(4) Which coefficient becomes zero for an even function ?

(b) Solve any one : **2**

(1) Write Fourier series of a function $f(x)$.

Given : $a_0 = \frac{\pi^2}{3}$, $a_n = \frac{4\cos n\pi}{n^2}$ and $b_n = 0$.

(2) A periodic function $f(x)$ with period 2π is defined as,

$$f(x) = x^2 \quad \text{for} \quad -\pi < x < \pi$$

Determine the Fourier coefficient a_0.

(c) Answer any one as directed : **3**

(1) Expand the following function in a Fourier series in the interval $[-\pi, \pi]$.

$$f(x) = -\left(\frac{(\pi + x)}{2}\right) \quad \text{for} \quad -\pi < x < 0$$

$$= \frac{\pi - x}{2} \quad \text{for} \quad 0 < x < \pi$$

(2) Deduce Fourier cosine series in interval 2π.

(d) Answer any one in detail : **5**

(1) Write Fourier series for a function with period $2l$, and obtain sine and cosine series in interval $(-l, l)$.

(2) Obtain Fourier series for a full wave rectifier function.

2 (a) Answer in brief : **4**

(1) On the base of __________ the constraints holonomic and non-holonomic are classified.

(2) How many degrees of freedom of a simple pendulum ?

(3) How many degrees of freedom of $2N$ particles with N equations of constraints in space ?

(4) Write the equation of D'Alembert's principle.

(b) Solve any one : **2**

(1) For a simple harmonic oscillator, kinetic energy $T = \frac{1}{2} m\dot{y}^2$ and potential energy $V = \frac{1}{2} m\omega^2 y^2$.

Find the Lagrange's equations of motion.

(2) The Lagrangian of an electric circuit comprising an inductance L and capacitance C is

$$L_E = \frac{1}{2} L\dot{q}^2 - \frac{1}{2}\frac{q^2}{C}$$

Take q as the generalized coordinate and obtain Lagrange's equations of motion.

(c) Answer any one as directed : **3**

(1) Deduce Lagrange's equations of motion for a spherical pendulum whose kinetic energy $T = \frac{1}{2} m \left(l^2\dot{\theta}^2 + l^2 \sin^2 \theta\dot{\varphi}^2\right)$ and potential energy $V = mgl\cos\theta$.

(2) Derive Lagrange's equations of motion from Hamilton's principle using $L = T - V$.

(d) Answer any one in detail : **5**

(1) Discuss generalized coordinates of simple pendulum, flywheel, bead on abacus, particle on the surface of a sphere and particle on a cone.

(2) Derive Newton's second law of motion from Hamilton's principle.

3 (a) Answer in brief : **4**

(1) The equation of motion of a simple pendulum is

$\ddot{\theta} + \frac{g}{l}\theta = 0$, what will be its period ?

(2) $\frac{\partial L}{\partial q_j} = 0$, then q_j is referred to as _________.

(3) Write Hamilton's modified principle.

(4) Write the expression of Lagrangian of a charged particle in an electromagnetic field.

(b) Solve any one : **2**

(1) Find Hamiltonian for the Lagrangian

$$L = \frac{\dot{x}^2}{2} - \frac{\omega^2 x^2}{2} - \alpha x^3 + \beta x \dot{x}^2$$

(2) The Hamiltonian of a system is given as

$$H = 2ml\left(p_z - ml\dot{\theta}\sin\theta\right)\sin\theta + \left(\frac{p_z - m\left(\dot{\theta}\sin\theta\right)^2}{2}\right) - \frac{ml^2\dot{\theta}^2}{2} - mgl\cos\theta - mgz$$

Obtain its Hamilton's equations.

(c) Answer any one as directed : **3**

(1) A particle moving near the surface of earth. The kinetic energy and potential energy of the particle are $T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$ and $V = mgz$.

Determine Hamilton's equations of motion for such conservative system.

(2) Describe phase space.

(d) Answer any one in detail : **5**

(1) Solve the problem of spherical pendulum using Lagrange's multiplier method.

(2) Obtain Hamilton's canonical equations from a variational principle.

4 (a) Answer in brief : **4**

(1) In photoelectric effect the numbers of photoelectrons emitted per second by the surface is directly proportional to the ________ of the incident radiation.

(2) With which name the expression $\Delta p \times \Delta x = \frac{\hbar}{2}$ is known ?

(3) In quantum mechanics, the difference $\alpha\beta - \beta\alpha$, of two dynamic variable α and β is called __________.

(4) In equation $A\psi(X) = a\psi(X)$, what is called a ?

(b) Solve any one : **2**

(1) Find A of a wave function $\psi(x) = Ae^{ikx}$ if it is normalized over a region $-a \le x \le a$.

(2) What kind of a wave function is a solution of the equation $-\frac{\hbar}{2\pi i}\frac{\partial\psi}{\partial t} = E\psi$?

(c) Answer any one as directed : **3**

(1) Find the expectation value $<X>$ of the position of a particle trapped in a box L wide. The wave function of the particle is $\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$.

(2) Explain experimental study of Compton effect.

(d) Answer any one in detail : **5**

(1) Derive one dimensional time dependent Schrödinger equation for free particle.

(2) Explain the particle in a three dimensional box.

5 (a) Answer in brief : **4**

(1) Fill up the blank : $L_z = -i\hbar(\_\_\_\_\_)$..

(2) In a two body problem, for an equation

$$\frac{-\hbar^2}{2\mu} \nabla^2{}_{red} \psi_{red} = E'' \psi_{red}$$

What ψ_{red} describe ?

(3) Which way a ket vector symbolized ?

(4) $<A|A> = 1$, then $|A>$ is _________.

(b) Solve any one : **2**

(1) Prove that $[X, Y] = -[Y, X]$.

(2) If $H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2$ then prove that $[x, H] = \frac{i\hbar p}{m}$.

(c) Answer any one as directed : **3**

(1) Two waves may be specified by $\psi_1 = \sin(ax + bt)$ and $\psi_2 = \sin(ax - bt)$. Show that $\psi = \psi_1 + \psi_2$ represents standing wave. Locate the nodes in it.

(2) Obtain Schrodinger wave equation for the oscillator.

(d) Answer any one in detail : **5**

(1) Describe angular momentum operator.

(2) Explain bra and ket vectors.